Homework 2

VU Wireless Communications 1, 389.157, SS 2015,
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Important for getting a grade:

- Answer all questions tagged with boxes such as XY short and precise, and state the question number next to the solution.
- Put the homework into the box located at the 1st floor of the *Institute of Telecommunication*, or send it to jkaitovi@nt.tuwien.ac.at.
- Attend the exercise lecture and be prepared to be called to the blackboard for presenting your results.
- In case questions arise, do not hesitate to contact me!

1 Rayleigh Fading Distribution

Starting from a Rayleigh distribution, with probability density function:

$$p(r) = \begin{cases} \frac{r}{\alpha} \exp\left(-\frac{r^2}{2\alpha}\right) & ; \quad r \geq 0 \\ 0 & ; \quad \text{else} \end{cases}, \tag{1}$$

where r is the instantaneous magnitude of the received voltage signal before envelope detection, and σ^2 it the time-average power of the received signal before envelope detection.

2 p [1] Find the probability that the envelope of the received signal doesn't exceed a specified value R.

3 p [2] Find the mean value of the Rayleigh distribution. (Hint: Use the Gaussian integral $\int_0^\infty \exp\left(-x^2\right) dx = \frac{\sqrt{\pi}}{2}$.)

3 p [3] Find the variance of the Rayleigh distribution.

1 p [4] Find the rms value of the envelope.

2 p [5] Find the median value of r.

The *level crossing rate* (LCR) is defined as the expected rate at which the Rayleigh fading envelope, normalized to the local rms signal level, crosses a specified level in a positive-going direction.
The number of level crossings per second is given by:

$$N_R = \int \frac{dr(t_1)}{dt} p\left(R = r(t_1), \frac{dr(t_1)}{dt}\right) d\left(\frac{dr(t_1)}{dt}\right) = \sqrt{2\pi}\nu_{\max}\rho e^{-\rho^2}, \qquad (2)$$

where $\nu_{\max}$ is the maximum Doppler frequency and $\rho = \frac{R}{R_{\text{rms}}}$ is the value of the specified level R, normalized to the local rms amplitude of the fading envelope.

The *average fade duration* is defined as the average period of time for which the received signal is below a specified level R:

$$\bar{\tau} = \frac{P_r\left[r \leq R\right]}{N_R}, \qquad (3)$$

and $P_r\left[r \leq R\right]$ is the probability that the received signal r is less than R.

4 p [6] A vehicle receives a 2.1 GHz transmission while travelling at a constant velocity for 10 s. The average fade duration for a signal level 10 dB below the rms level is 1 ms. How far does the vehicle travel during the 10 s interval?

3 p [7] How many fades does the signal from Question [6] undergo at the rms threshold level during a 10 s interval? Assume that the local mean remains constant during the travel.

2 Moving Users and Doppler Spread

2 p [8] Users of a communication system at 2.1 GHz are moving with 150 km/h. Find $\nu_{\max}$ and the coherence time. (Hint: use the Fleury uncertainty relationship $T_{\text{coh}} \leq \frac{1}{2\pi S_\nu}$, and $S_\nu = \frac{\nu_{\max}}{\sqrt{2}}$).

1 p [9] How does the coherence time influence a communication system?

1 p [10] Looking at the Fleury uncertainty relationship, explain how the rms Doppler spread influences the coherence time.

2 p [11] Assume that one pilot symbol is enough to estimate the channel correctly for a duration of T_{coh}. How many symbols in one frame have to be pilot symbols for channel tracking? Use the coherence time calculated in Question [8] and assume a frame duration of a)$T_f = 700\,\mu$s, b)$T_f = 7.5$ ms.

3 Moments of the Power Delay Profile

A local spatial average of a power delay profile $P_r(\tau)$ measured at 2.1 GHz is shown in Figure 1.

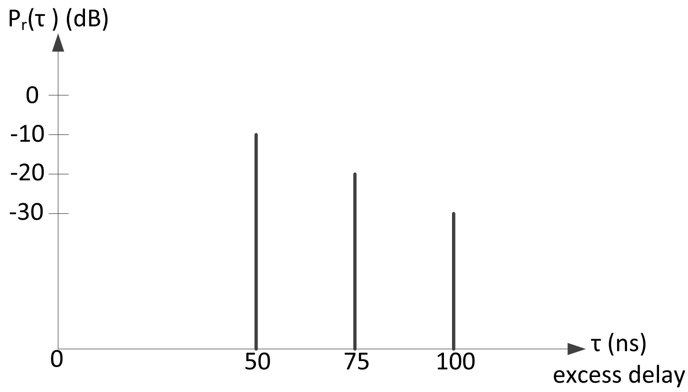

Figure 1: Indoor channel Response

1 p [12] Calculate the mean power P_m.

2 p [13] Find the mean delay T_m for this channel.

2 p [14] For this channel calculate the rms delay spread S_τ.

1 p [15] If a particular modulation provides suitable BER performance whenever $\frac{S_\tau}{T_s} \leq 0.1$, determine the shortest symbol period T_s that can be sent through the RF channel shown in Figure 1, without using an equalizer.

1 p [16] Determine the highest symbol rate that may be sent through the RF channel of Question [15].