**Guidelines**

- Write your name and matriculation number on each sheet of paper.
- Only clearly readable exercise-elaborations are evaluated.
- Results have to be provided together with an evident way of calculation.
- Keep textual answers short and concise. Lengthy or vague statements won't gain points.

**Exercise 1.1 (0.5 points)**

*The bilinear transform is a widely used method for designing digital filters from an already designed analog filter. It can be interpreted as a mathematical transformation from the s-domain (laplace transform) to the z-domain (z-transform):*

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\frac{z-1}{z+1}}.$$

*We first investigate on how the transformation maps certain regions of the s-plane onto the z-plane.*

1. *Calculate $r$ and $\phi$ of the term $z = re^{j\phi}$ in terms of the real part $\sigma$ and the imaginary part $\omega$ of $s = \sigma + j\omega$.*
2. *Consider the general value $s = \sigma + j\omega$. Determine the corresponding mappings from the s-plane to the z-plane for $\sigma > 0, \sigma = 0$ and $\sigma < 0$, respectively (which regions of the s-plane are mapped onto which regions of the z-plane?). Display your findings by appropriate sketches.*
3. *For $\sigma = 0$, $s$ becomes $s = j\omega$. Determine the relation between the frequencies $\omega$ in the analog system and the frequencies $\Omega$ in the digital system.*
4. *Given the transfer function of an analog filter*

$$H(s) = \frac{s+2}{(s+1)(s+3)}, \tag{1}$$

*design the corresponding digital filter, by applying the bilinear transform. Determine*

(a) *the corresponding transfer function $H(z)$,*

(b) *the discrete-time impulse response $h[n]$.*

*Hint: Employ a partial fraction expansion.*

**Exercise 1.2 (0.5 points)**

1. *Show that each system with transfer function*

$$H(e^{j\Omega}) = A(e^{j\Omega})e^{-j\alpha\Omega+j\beta} \tag{2}$$

*and real-valued* $A(e^{j\Omega})$ *has linear phase and constant group delay. Explicitly calculate phase and group delay of such a system.*

2. *In literature we distinguish between four different types of FIR filters with linear phase, whose impulse responses* $h_n$ *are defined by:*

   *(a)* $h_n = h_{M-n} \quad 0 \leq n \leq M \quad M$ *even*

   *(b)* $h_n = h_{M-n} \quad 0 \leq n \leq M \quad M$ *odd*

   *(c)* $h_n = -h_{M-n} \quad 0 \leq n \leq M \quad M$ *even*

   *(d)* $h_n = -h_{M-n} \quad 0 \leq n \leq M \quad M$ *odd.*

   *For each of the four systems, sketch an exemplary impulse response (For this task choose* $M = 5$ *and* $M = 6$*, respectively).*

3. *Now assume four general* $h_n$*, which satisfy the properties of the four filter types defined above. Calculate the four resulting transfer functions for an arbitrary* $M$ *(*$M$ *can be assumed to be even or odd, according to the used filter type). Further, determine phase and group delay for each filter type.*

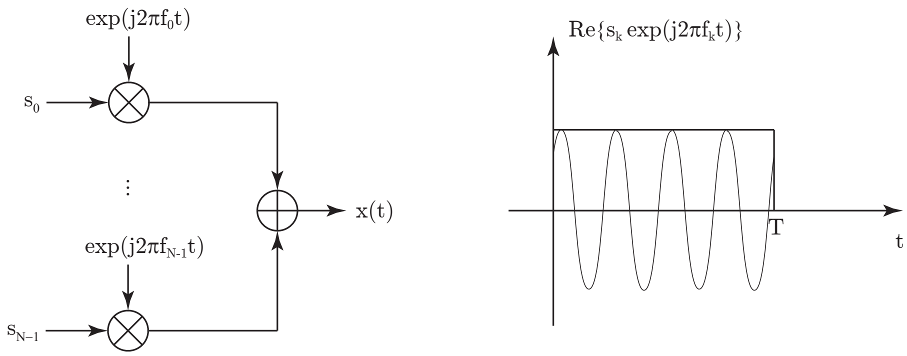


Figure 1.1: Signal modulation

**Exercise 1.3 (0.5 points)**

*A stream of complex valued symbols is split into blocks of $N$ symbols each. The $N$ symbols of each block are modulated on $N$ carriers $f_k$ with constant frequency spacing $\Delta f$. The signal is windowed with a window length $T$, as indicated in Figure 1.1.*

1. *Let $s_k$ denote the $k$-th symbol of a block, with $k = 0, \ldots, N-1$. Provide a low-pass equivalent description of the signal $x(t)$, i.e., set $f_0 = 0$. Hint: Express $f_k$ in terms of $\Delta f$.*
2. *Let $\Delta f = \frac{1}{NT_s}$ and assume that the signal is sampled at $t = nT_s$, where $T_s$ denotes the sampling period. Provide a discrete time description $x[n]$ of the signal.*
3. *Given $T_s$, how do you have to choose $T$ such that the sampled signal is Intersymbol Interference (ISI) free within a block?*
   *Hint: Consider the Nyquist ISI criterion.*
4. *Which type of linear transformation is performed here on the symbols of each block?*
5. *Reformulate the discrete time description of $x[n]$ into vector-matrix notation, assuming $\underline{s} = [s_0, \ldots, s_{N-1}]^T$ and $\underline{x} = [x_0, \ldots, x_{N-1}]^T$ as the input- and output-vectors, respectively. Then, answer the following questions:*
   (a) *Are the columns of the resulting matrix linearly independent?*
   (b) *Are the columns of the resulting matrix mutually orthogonal?*
   *Hint: Consider the geometric series*

$$\sum_{n=0}^{N-1} e^{j2\pi(k-k')n/N} \tag{3}$$

(c) *Normalize the matrix such that each vector (i.e., column) in the matrix has length* 1.

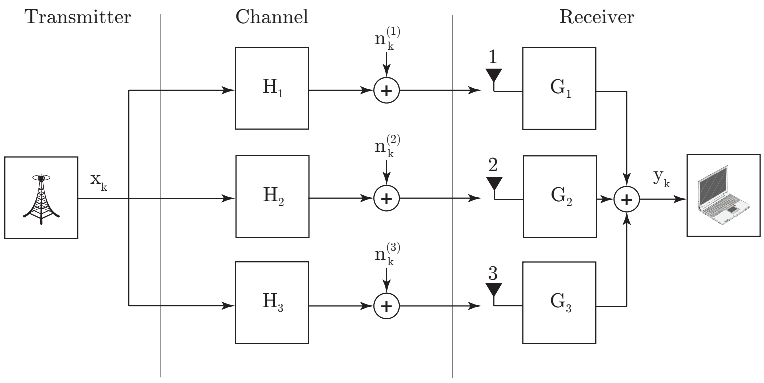


Figure 1.2

## MATLAB-Exercise 1.1 (1 points)

*Consider a SIMO (Single Input Multiple Output) experimental setup, as shown in Figure 1.2. The scenario is assumed to be static and the channels are known exactly from prior measurements:*

$$H_1(q^{-1}) = 1 - 0.25q^{-2} \tag{4}$$
$$H_2(q^{-1}) = 1 - q^{-1} + 0.25q^{-2} \tag{5}$$
$$H_3(q^{-1}) = 1 + 2q^{-1} \tag{6}$$

*The target of the experiment is to receive a distortion-free signal $y_k = x_{k-1}$ (where $x_k$ denotes the input signal). This should be achieved with as few antennas as possible. The corresponding receive filters for each antenna are given as*

$$G_i(q^{-1}) = g_0^{(i)} + g_1^{(i)}q^{-1} \tag{7}$$

1. ***Preliminaries***
   *In the first part of the exercise, the noise is assumed as $n_k^{(i)} = 0, \forall i$.*

   (a) *Use MATLAB to determine the filter coefficients $g_k^{(i)}$ for the cases:*
      i. *Only antenna 1 is used.*
      ii. *Antenna 1 and 2 are used.*
      iii. *All three antennas are used.*

      *If a calculation is not possible, reason your answer in detail. Note:*
      - *You can use the "\" operator in MATLAB (see documentation for further details)*
      - *Verify your results since MATLAB will generate output for all three cases.*

   (b) *Can one receive antenna be turned off without changing the output behavior of the system? Reason your answer.*

2. ***Simulations***

*In the second part of the exercise, the scenario as shown in Figure 1.2 is simulated. The target is to investigate the behavior of the system under various SNR (Signal-to-Noise-Ratio) conditions with the receive filters as obtained in the first part.*

*Preparation:*

- *Generate a stream of random symbols $x_k$ from the alphabet $\{1+j, -1+j, -1-j, 1-j\}$, i.e., each symbol codes for 2 bits. All symbols are equally likely and $|x_k|^2 = 1$.*
- *Generate a stream of properly normed noise samples for each channel. The samples are assumed to be white complex Gaussian.*

(a) *For an SNR of* $10\,\mathrm{dB}$

i. *Provide a scatterplot of the input signal of receive filter* 3.

ii. *Draw a scatterplot of the receive signal after filtering ($y_k$ in the figure).*

(b) *For an SNR-range* $-10\,\mathrm{dB}\ldots 20\,\mathrm{dB}$

i. *Decode the stream of received symbols by Maximum-Likelihood (ML) detection, i.e., the detected symbol is obtained as* $\mathrm{sign}(\Re(y_k)) + j\,\mathrm{sign}(\Im(y_k))$, *where* $\mathrm{sign}(\cdot)$ *denotes the sign function*

$$\mathrm{sign}(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0. \end{cases} \tag{8}$$

ii. *Provide a plot of the Bit Error Ratio (BER) versus Signal to Noise Ratio (SNR) in double-logarithmic scale (In order to get meaningful values for the BER, choose the number of simulated symbols sufficiently large).*