Figure 2.1: Resonant circuit

Guidelines

- Write your name and matriculation number on each sheet of paper.

- Only clearly readable exercise-elaborations are evaluated.

- Results have to be provided together with an evident way of calculation.

- Keep textual answers short and concise. Lengthy or vague statements won't gain points.

Exercise 2.1 (0.5 points)
Consider a resonant circuit as shown in Figure 2.1.

1. *Provide a state space description of the circuit in matrix-vector notation, where $V_C(t) = V_O(t)$ and $I_L(t)$ are the states of the system, and $V_O(t)$ and $V_I(t)$ denote the input- and output voltage, respectively.*

2. *Use the state space description to derive the transfer function $H(s) = \frac{V_0(s)}{V_I(s)}$.*

3. *Determine the state space description in companion form and sketch the corresponding Infinite Impulse Response (IIR) filter.*

4. *Calculate the impulse response $h(t)$ of the filter.*

5. *Given $R = 100\,\Omega$, $C = 1\,\mathrm{F}$ and $L = 1\,\mathrm{H}$. Is the system stable?*

S. Schwarz, M. Taranetz, M. Rupp

Exercise 2.2 (0.5 points)
Consider a causal *and* stable *LTI system with* real-valued *impulse response h_n. The real part of the corresponding transfer function is given as:*

$$H_R(e^{j\Omega}) = \Re\left\{H\left(e^{j\Omega}\right)\right\} = \frac{1 + a\cos(\Omega)}{1 + 2a\cos(\Omega) + a^2}, \qquad \text{with } |a| < 1 \qquad (1)$$

1. *Calculate the even part $h_n^{(e)}$ of the impulse response h_n.*
   Hint:

$$x_{n+n_0} \leftrightarrow z^{n_0} X(z) \qquad\qquad\qquad (2)$$
$$\delta_n \leftrightarrow 1 \qquad\qquad \forall z \qquad\qquad (3)$$
$$\alpha^n u_n \leftrightarrow \frac{z}{z - \alpha} \qquad\qquad |z| > |\alpha| \qquad\qquad (4)$$
$$-\alpha^n u_{-n-1} \leftrightarrow \frac{z}{z - \alpha} \qquad\qquad |z| < |\alpha| \qquad\qquad (5)$$

   where u_n is the unit step function.

2. *From $h_n^{(e)}$, determine the odd part $h_n^{(o)}$ of the impulse response.*

3. *Calculate $H\left(e^{j\Omega}\right)$.*

S. Schwarz, M. Taranetz, M. Rupp

Exercise 2.3 (0.5 points)
Consider the following system

$$H(z) = \frac{12z^2 + 23z - 2}{12z^2 - z - 1}. \tag{6}$$

1. *Calculate a partial fraction decomposition of the transfer function $H(z)$.*

2. *Calculate the poles and zeros of the system and sketch a pole-zero diagram.*

3. *State all possible regions of convergence (ROCs) and determine whether the corresponding systems are causal/acausal and stable/unstable.*

4. *Derive and sketch the impulse response of the stable system.*

5. *Determine the state space description of the causal filter in companion-form and draw a signal-flow diagram indicating the positions of the input-, output- and state-signals.*

6. *(**Extra 0.25 points**) Determine stability of the system using the state-space description.*

Hint: Employ the Z-transform.

$$\delta_k \Leftrightarrow 1 \tag{7}$$

$$c^k \sigma_k \Leftrightarrow \frac{1}{1 - c\,z^{-1}}, \ |z| > |c| \tag{8}$$
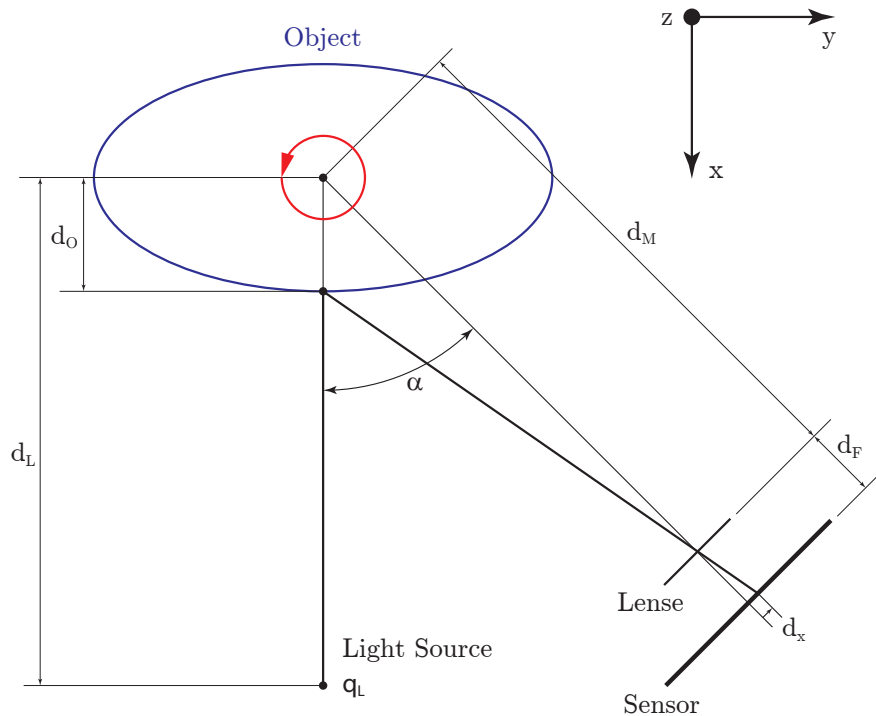
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Figure 2.2: 3D scanner system

MATLAB-Exercise 2.1 (1.5 points)

*In this MATLAB exercise, the basic principles of 3D-scanning and pattern recognition are investigated (http://www.youtube.com/watch?v=RVgyyIlQydg).
Consider a 3D-scanning system as shown in Figure 2.2. The object is placed on a rotation stage. For each measurement, a light source q_L is moved in z-direction to scan the object vertically. The deviation d_x of the light-ray is captured by a sensor. Then, the object is rotated by a fixed angle Δ_M and the next measurement is performed.*

1. *Given the parameters $\{\alpha, d_M\ d_F\}$, calculate the function $d_O(d_x)$*

2. *Your next task is to reconstruct and plot a given measurement.
   Download the file* `dx_measurement.mat` *from our homepage. The file includes a measurement matrix. Each column corresponds to the measurements of d_x in vertical direction, which are equally spaced from $z = -1$ cm to $z = 1$ cm. The number of columns corresponds to the number of measured angle position during a full rotation by 2π. The following parameters were applied in the loaded measurement*

$$\alpha = 20° \tag{9}$$

$$d_M = 1 \text{ m} \tag{10}$$

$$d_F = 0.001 \text{ m.} \tag{11}$$

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(a) *From the measurement, reconstruct the object by generating three matrices X,Y,Z, corresponding to the surface coordinates of the object. The columns of each matrix should refer to one measurement in vertical direction.*

(b) *Provide a 3-dimensional plot of the object, by using e.g. the MATLAB commands* `surf` *or* `meshgrid`.

3. *A common technique to recognize the object is to compare its scan-data to scan-data of known objects in the Fourier domain. Therefore, a two-dimensional Discrete Fourier-transform (DFT), which is defined as*

$$\mathcal{A}[k,l] = \frac{1}{\sqrt{MN}} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} a[m,n] e^{-2\pi i \frac{mk}{M}} e^{-2\pi i \frac{nl}{N}} \qquad (12)$$

is applied on the scan-data. Your task is to compute the two-dimensional Discrete Fourier Transform (DFT) of the scan data without using MATLAB's built in functions. Therefore, note that the DFT is separable, i.e., Equation (12) can be rewritten in the form of two successive one-dimensional DFTs. This property can be exploited to formulate Equation (12) in vector matrix notation as

$$A = W_M \Lambda W_N \qquad (13)$$

where $A_{k,l} = \mathcal{A}[k,l]$, $\Lambda_{m,n} = a[m,n]$ and W_M and W_N denote the Fourier transformation matrices of dimension $M \times M$ and $N \times N$, respectively.

(a) *Determine the W_M and W_N, respectively.*

(b) *Use MATLAB to calculate A and plot its absolute value (e.g., with* `imagesc`*). In our case, the matrix Λ is the scan-data.*

(c) *Can you make a statement about the symmetry of the object from the Fourier-transformed data? If yes, which? If no, why?*

S. Schwarz, M. Taranetz, M. Rupp