# 2 Basic Math, Part 2

**Exercise 2.1**
*Write the vector* $\underline{v} = [1,-2,5]$ *in* $\mathcal{R}^3$ *in form of a linear combination of the vectors* $\underline{e}_1 = [1,1,1]$, $\underline{e}_2 = [1,2,3]$, $\underline{e}_3 = [2,-1,1]$.

Solution: $\underline{v} = -6\underline{e}_1 + 3\underline{e}_2 + 2\underline{e}_3$.

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**Exercise 2.2**
*Write the vector* $\underline{v} = [2,-5,3]$ *in* $\mathcal{R}^3$ *in form of a linear combination of the vectors* $\underline{e}_1 = [1,-3,2]$, $\underline{e}_2 = [2,-4,-1]$, $\underline{e}_3 = [1,-5,7]$.

Solution: The according system of equations is inconsistent and thus has no solution. The vector $\underline{v}$ does not lie in the space spanned by the vectors $\underline{e}_1$, $\underline{e}_2$ and $\underline{e}_3$.

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**Exercise 2.3**
*Show that the vectors* $\underline{u} = [1,2,3]$, $\underline{v} = [0,1,2]$ *and* $\underline{u} = [0,0,1]$ *span the space* $\mathcal{R}^3$.

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**Exercise 2.4**
*Evaluate if the following vectors defined in* $\mathcal{R}^3$ *are linear dependent:*

*a.* $[1,-2,1]$, $[2,1,-1]$, $[7,-4,1]$

*b.* $[1,-3,7]$, $[2,0,-6]$, $[3,-1,-1]$, $[2,4,-5]$

*c.* $[1,2,-3]$, $[1,-3,2]$, $[2,-1,5]$

*d.* $[2,-3,7]$, $[0,0,0]$, $[3,-1,-4]$

Solutions:

a. linear dependent

b. linear dependent

c. linear independent

d. linear dependent

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**Exercise 2.5**

*For each of the following operators $T : \mathcal{R}^3 \to \mathcal{R}^3$, derive all eigenvalues and a basis for the eigenspace:*

a. $T(x,y,z) = (x + y + z, 2y + z, 2y + 3z)$

b. $T(x,y,z) = (x + y, y + z, -2y - z)$

c. $T(x,y,z) = (x - y, 2x + 3y + 2z, x + y + 2z)$

Solutions:

a. $\lambda_1 = 1$, $\underline{u} = [1,0,0]$; $\lambda_2 = 4$, $\underline{v} = [1,1,2]$

b. $\lambda_1 = 1$, $\underline{u} = [1,0,0]$. There are no other eigenvalues in $\mathcal{R}$

c. $\lambda_1 = 1$, $\underline{u} = [1,0,-1]$; $\lambda_2 = 2$, $\underline{v} = [2,-2,-1]$; $\lambda_3 = 3$, $\underline{w} = [1,-2,-1]$

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**Exercise 2.6**

*Evaluate the Jordan normal form of*

*1.*

$$\begin{bmatrix} 4 & 1 & 3 \\ 0 & 4 & 1 \\ 0 & 0 & 4 \end{bmatrix}$$

*2.*

$$\begin{bmatrix} 3 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

Solutions:

1.

$$\begin{bmatrix} 4 & 1 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 4 \end{bmatrix}$$

2.

$$\begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

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**Exercise 2.7**

*With the help of the Gram-Schmidt procedure to orthogonalize, evaluate a orthonormal basis of the subspace* $W$ *of* $\mathcal{C}^3$*, which is spanned by* $\underline{v}_1 = [1,i,0]$ *and* $\underline{v}_2 = [1,2,1-i]$*.*

Solution: $\underline{u}_1 = \left[\frac{1}{\sqrt{2}},\frac{i}{\sqrt{2}},0\right]$ ; $\underline{u}_2 = \left[\frac{1+2i}{\sqrt{18}},\frac{2-i}{\sqrt{18}},\frac{2-2i}{\sqrt{18}}\right]$