Guidelines

- Write your name and matriculation number on each sheet of paper.
- Only clearly readable exercise-elaborations are evaluated.
- Results have to be provided together with an evident way of calculation.
- Keep textual answers short and concise. Lengthy or vague statements won't gain points.

Exercise 2.1 (0.5 points)
Consider a *maximum phase system*, whose z-transform has all zeroes *outside* of the unit circle. A finite causal maximum phase sequence with a given amplitude function of its Fourier transform can be achieved by mirroring all zeros of the z-transform of a minimum phase sequence to the conjugate reciprocal positions outside of the unit circle. That means we can express the z-transform of a finite causal maximum phase sequence as

$$H_{\texttt{max}}(z) = H_{\texttt{min}}(z) H_{\texttt{ap}}(z).$$

Doing so ensures that $|H_{\texttt{max}}(e^{j\omega})| = |H_{\texttt{min}}(e^{j\omega})|$. Now the z-transform of a finite minimum phase sequence can be expressed as

$$H_{\texttt{min}}(z) = h_{\texttt{min},0} \prod_{k=1}^{M} (1 - c_k z^{-1}), \qquad |c_k| < 1.$$

1. Find the expression for the allpass system function that mirrors all zeroes of $H_{\texttt{min}}(z)$ to positions outside of the unit circle.
2. Show that $H_{\texttt{max}}(z)$ can be written as (keep in mind that we consider real-valued signals and therefore also real-valued filter coefficients!)

   $$H_{\texttt{max}}(z) = z^{-M} H_{\texttt{min}}(z^{-1}).$$

3. With the previously attained results, express the impulse response $h_{\texttt{max},n}$ of the maximum phase system in terms of the impulse response $h_{\texttt{min},n}$ of the minimum phase system.

Terms: Maximum phase, z-transform.

Exercise 2.2 (0.5 points)
Consider a system with transfer function

$$A(\omega) = \begin{cases} \frac{\alpha}{B}\omega + A_0 & ; \ |\omega| \le B \\ 0 & \text{else} \end{cases} \quad , A_0, B, \alpha \in \mathbb{R}.$$

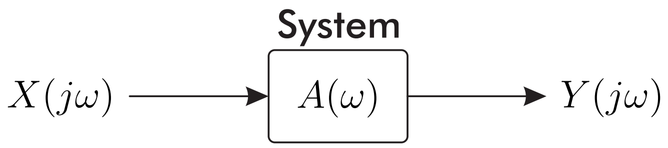

Figure 2.1

1. Draw $A(\omega)$ in frequency domain for the two cases of $\alpha = \{0{,}0.5\}$.
2. Determine the corresponding time-domain representation $a(t) = \mathcal{F}^{-1}\{A(\omega)\}$ via inverse Fourier-transform. Use the correspondences (also see Figure 2.2)

$$\frac{d^n}{dt^n} f(t) \stackrel{\mathcal{F}}{\Longleftrightarrow} (j\omega)^n X(\omega),$$

$$\text{sinc}\left(\frac{\pi}{T_s} t\right) \stackrel{\mathcal{F}}{\Longleftrightarrow} T_s \text{rect}\left(\omega; \frac{\pi}{T_s}\right).$$

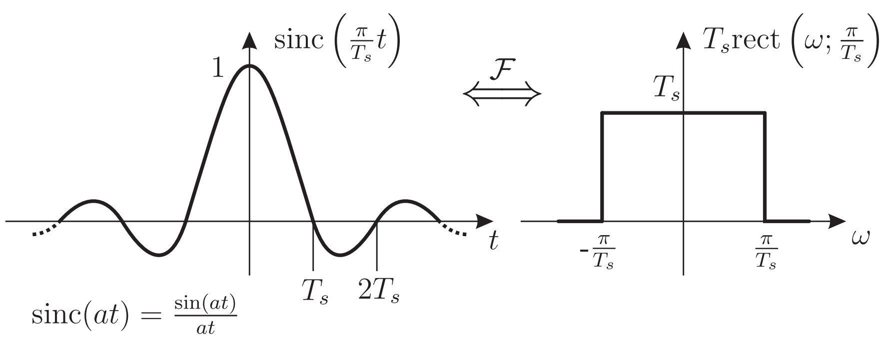

Figure 2.2: Correspondence of sinc() and rect(;).

3. Is the system linear/time-invariant/causal in general?

We now introduce an input signal in time domain as

$$x(t) = \cos(\omega_1 t) \ , \ \omega_1 = \frac{B}{2}.$$

4. Compute the output signal in frequency- and time-domain, i.e., $Y(j\omega)$ and $y(t)$.

5. For which value(s) of α is the output $y(t)$ an undistorted cosine?

6. Let $\overline{y}(t)$ be the output signal in case of $\alpha = 0$. Compute the mean squared error between $y(t)$ and $\overline{y}(t)$ as function of α:

$$\mathrm{MSE}(\alpha) = \frac{1}{T}\int_0^T |y(t) - \overline{y}(t)|^2 dt,$$

note that $\omega_1 = \frac{2\pi}{T}$.

Terms: Fourier-transform.

Exercise 2.3 (0.5 points)

1. Show that each system with transfer function

$$H(e^{j\Omega}) = A(e^{j\Omega})e^{-j\alpha\Omega + j\beta},$$

where $A(e^{j\Omega})$ is real valued, has linear phase and constant group delay. Explicitly calculate phase and group delay of such a system.

2. In literature we distinguish between four different types of FIR filters with linear phase:

(a) $h_n = h_{M-n} \quad 0 \le n \le M \quad M$ even
(b) $h_n = h_{M-n} \quad 0 \le n \le M \quad M$ odd
(c) $h_n = -h_{M-n} \quad 0 \le n \le M \quad M$ even
(d) $h_n = -h_{M-n} \quad 0 \le n \le M \quad M$ odd.

For each of the four systems, sketch an exemplary impulse response (with $M = 5$ and $M = 6$, respectively).

3. Now calculate the transfer functions for arbitrary h_n which satisfy the properties of the four filter types. Further, determine phase and group delay for each filter type.

Terms: Linear phase, group delay.

MATLAB-Exercise 2.1 (1 point)

Reconsider the maximum and minimum phase system from Exercise 2.1. Your task in this exercise is to confirm with the help of MATLAB, that these systems truly have the minimum and maximum phase for a given set of zeroes and its mirrored values.

The zeroes of a maximum phase system are defined as (the system has no poles)

$$\begin{bmatrix} -5+j & -5-j \\ -1+j5 & -1-j5 \\ 7+j4 & 7-j4 \end{bmatrix}$$

1. Plot the pole/zero diagram for this maximum- and the corresponding minimum phase system.
2. Plot the pole/zero diagram and the magnitude response of the allpass transforming the maximum- into a minimum phase system.
3. For each pair of conjugate complex zeroes, there are two possibilities: outside of the unit circle or mirrored to the inside of the unit circle. Plot the phase functions for all combinations. Check if the minimum- and maximum phase system truly show the minimal and maximal phase. Also plot the derivative of these phase functions.

Helpful MATLAB functions: freqz(b,a,n), phasez(b,a,n), zplane(b,a), poly(r), fvtool(b,a)

Terms: Minimum-, maximum phase system, pole/zero diagram.