Problem Set 10

Problem 10.1 Check if following discrete functions are valid autocorrelation functions:

a) $r[m] = m \cdot e^{-|m|}$

b) $r[m] = \cos(\theta_0 m)$

c) $r[m] = a^{|m|}$, for $|a| < 1$

d) $r[m] = \frac{1}{1+m^3}$

e) $r[m] = \begin{cases} 1, & \text{for} \quad |m| \leq N_1 \\ 0, & \text{for} \quad |m| > N_1 \end{cases}$

Find the mean power for those cases where $r[m]$ is a valid autocorrelation function.

Problem 10.2 Let $\mathsf{z}[n] = \mathsf{x}\cos(\theta_0 n) + \mathsf{y}\sin(\theta_0 n)$, where x and y are two uncorrelated zero-mean random variables.

a) Find the mean and the autocorrelation function of $\mathsf{z}[n]$.

b) Which condition must x and y fulfill if we want $\mathsf{z}[n]$ to be a wide-sense stationary process?

c) Find the mean power and power spectral density (PSD) of the stationary process from the previous part and sketch the PSD.

Problem 10.3 Consider the real random process

$$\mathsf{x}[n] = e^{-\mathsf{a}n},$$

where a is a random variable, uniformly distributed in the interval $[0, 1]$.

a) Calculate the mean of $\mathsf{x}[n]$.

b) Calculate the autocorrelation function of $\mathsf{x}[n]$.

c) Is $\mathsf{x}[n]$ stationary or wide-sense stationary?

Problem 10.4 Consider a random process $\mathsf{x}[n] = \mathsf{v}[n]+\mathsf{z}[n]$. Here, $\mathsf{v}[n] = \mathsf{a}\cos(\theta_0 n+\Phi)$ is a stationary process with a constant frequency θ_0, a random amplitude a uniformly distributed in the interval $[0, a_0]$, and a random phase Φ uniformly distributed in the interval $[-\pi, \pi[$. The random variables a and Φ are statistically independent. The random process $\mathsf{z}[n]$ is zero-mean, stationary, white, and statistically independent of $\mathsf{v}[n]$, with power spectral density $S_{\mathsf{n}}(e^{j\theta}) = \nu/2$.

a) Calculate the mean, autocorrelation function, and power spectral density of $\mathsf{x}[n]$.

b) Suppose that $\mathsf{x}[n]$ is the input of an ideal bandpass filter with carrier frequency θ_c and bandwidth $2\theta_{BW}$. Calculate the mean power of the output signal $\mathsf{y}[n]$ (distinguish between the two cases: θ_0 in the pass band and θ_0 outside the pass band).